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Strategic Asset Allocation Revisited

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Abstract

This paper studies the strategic asset allocation problem of constant relative risk averse investors. We propose a parametric linear portfolio policy that accommodates an arbitrarily large number of assets in the portfolio and state variables in the information set. Our method is made operational through an overidentified system of first order conditions of the maximization problem that allows us to apply GMM for parameter estimation and several specification tests. The empirical results for a portfolio of stocks, bonds and cash provide ample support to the linear specification of the portfolio weights and reveal significant differences between myopic and strategic optimal portfolio allocations.

Key words: dynamic hedging demand; intertemporal portfolio theory; parametric portfolio policy rules; return predictability; strategic asset allocation.

JEL Codes: E32, E52, E62.

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1 Introduction

Optimal portfolio decisions depend on the details of the economic and financial environment: the financial assets that are available, their expected returns and risks, and the preferences and circumstances of investors. These details become particularly relevant for long-term investors. Such investors must concern themselves not only with expected returns and risks today, but with the way in which expected returns and risks may change over time. It is widely understood at least since the work of Samuelson (1969) and Merton (1969, 1971, 1973) that the solution to a multiperiod portfolio choice problem can be very different from the solution to a static portfolio choice problem. In particular, if investment opportunities vary over time, then long-term investors care about shocks to investment opportunities as well as shocks to wealth itself. This can give rise to intertemporal hedging demands for financial assets and lead to strategic asset allocation as a result of the farsighted response of investors to time-varying investment opportunities.

Unfortunately, intertemporal asset allocation models are hard to solve in closed form unless strong assumptions on the investor's objective function or the statistical distribution of asset returns are imposed. A notable exception is when investors exhibit log utility with constant relative risk aversion equal to one. This case is relatively uninteresting because it implies that Merton's model reduces to the static model. Another exception within the class of utility functions describing constant relative risk aversion and represented by the family of power utility functions is when asset returns are log-normally distributed. In this case, maximizing expected utility is equal to the mean-variance analysis proposed by Markowitz (1952) in his seminal study. In this model, the investor trades off mean against variance in the portfolio return. The relevant mean return is the arithmetic mean return and the investor trades the log of this mean linearly against the variance of the log return. The coefficient of relative risk aversion acts as a penalty term adding to the variance of the return.

More generally, the lack of closed-form solutions for optimal portfolios with constant relative risk aversion has limited the applicability of the Merton model and has not displaced the Markowitz model. This situation has begun to change as a result of several developments in numerical methods and continuous time finance models. More specifically, some authors such as Balduzzi and Lynch (1999), Barberis (2000), Brennan et al. (1997, 1999) and Lynch (2001) provide discrete-state numerical algorithms to approximate the solution of the portfolio problem over infinite horizons. Closed-form solutions to the Merton model are derived in a continuous time model with a constant risk-free interest rate and a single risky asset if long-lived investors have power utility defined over terminal wealth (Kim and Omberg, 1996), or if investors have power utility defined over consumption (Wachter, 2002), or if the investor has Epstein and Zin (1989, 1991) utility with intertemporal elasticity of substitution equal to one (Campbell and Viceira, 1999; Schroder and Skiadas, 1999). Approximate analytical solutions to the Merton model have been developed in Campbell and Viceira (1999, 2001, 2002) and Campbell et al. (2003) for models exhibiting an intertemporal elasticity of substitution not too far from one.

A recent alternative to solving the investor's optimal portfolio problem has been proposed by Brandt (1999), Aït-Sahalia and Brandt (2001) and Brandt and Santa-Clara (2006). Aït-Sahalia and Brandt (2001), for example, show how to select and combine variables to best predict the optimal portfolio weights, both in single-period and multiperiod contexts. Rather than first model the various features of the conditional return distribution and subsequently characterize the portfolio choice, these authors focus directly on the dependence of the portfolio weights on the predictors. They do this by solving sample analogues of the conditional Euler equations that characterize the portfolio choice, as originally suggested by Brandt (1999). Brandt and Santa-Clara (2006) solve the dynamic portfolio selection problem by expanding the asset space to include mechanically managed portfolios and compute the optimal static portfolio within this extended asset space. The intuition of this strategy is that a static choice of managed portfolios is equivalent to a dynamic strategy.

The main contribution of this paper is to propose a simple framework to study the optimal asset allocation problem of an investor exhibiting constant relative risk aversion (CRRA) over long, potentially infinite, horizons. Our model accommodates an arbitrarily large number of assets in the portfolio and state variables in the information set. In contrast to most of the related literature, our approach relies on the first order conditions of the maximization problem that yield a tractable system of equations that can be solved using standard econometric methods. To do this we entertain the same optimal linear portfolio policy rule proposed by Aït-Sahalia and Brandt (2001) to describe, in our framework, the dynamics of the portfolio weights over the investor's multiperiod horizon. The main advantage of our approach is that the first order conditions of the maximization problem yield a simple system of equations that is overidentified and provides a very intuitive empirical representation.

The second contribution of the paper is to exploit this feature of the model to estimate the model parameters and make suitable inference. More specifically, we apply the generalized method of moments (GMM) of Hansen and Singleton (1982) for estimation and also for deriving different specification tests. In particular, to the best of our knowledge, this is the first paper to propose a testing framework for the parametric portfolio policy specification. We do this by developing two different but related tests. First, we adapt the specification J-test obtained from the overidentified system of equations to assess whether the linear parametric portfolio policy proposed, for example, in Aït-Sahalia and Brandt (2001) and Brandt and Santa-Clara (2006), is statistically correctly specified for long investment horizons. Second, we adapt the incremental testing approach developed by Sargan (1958, 1959) to assess the marginal statistical relevance of the state variables in the linear portfolio policy specification. We complete the econometric section by proposing a test that gauges the effect of the number of investment horizons on the optimal allocation of assets to the portfolio. This is done by developing a Hausman type test that compares different specifications of the investor's maximization problem in terms of the investment horizon. More specifically, we contemplate a short-term and a long-term investment horizon and compare the informational content of the period spanning between the short and long-term horizons. Under the null hypothesis the informational content of this period is null implying that the parameter estimators obtained imposing the null hypothesis are consistent estimates of the population parameters driving the optimal portfolio weights.

The third contribution is empirical and consists of comparing the optimal allocation of assets of a myopic investor only concerned with maximizing one period ahead wealth with the allocation of a strategic investor with a long multiperiod investment horizon. The empirical application closely follows similar studies such as Brennan et al. (1997), Brandt (1999) and Campbell et al. (2003). The investor is assumed to invest in a portfolio given by three assets - a one-month Treasury bill as riskless security, a long-term bond, and an equity portfolio. The variables that predict expected returns on these assets are the detrended short-term interest rate, the U.S. credit spread, the S&P 500 trend and the one-month average of excess stock and bond returns. Our econometric specification shows that the strategic allocation to the S&P 500 and bond indices differs in two main aspects with respect to the myopic asset allocation. First, the absolute value of the optimal portfolio weights in the strategic case is usually larger than in the myopic case. Second, the strategic allocation to the S&P 500 is found to be positively and significantly related to the trend variable and negatively to the detrended short-term interest rate whereas the strategic allocation to bonds is found to be negatively and significantly related to the detrended short-term interest rate as the degree of risk aversion increases. The analysis of the optimal stocks and bonds' hedging demand varies significantly with the state variables rather than being stable over time highlighting the importance of the dynamics of the state variables in determining the strategic optimal portfolio allocation and the differences with the myopic strategy. Our empirical analysis also suggests that the optimal allocation to bonds is larger in the strategic case than in the myopic one.

The closest contributions to our study are Brennan et al. (1997), Aït-Sahalia and Brandt (2001), Campbell and Viceira (2003), and more recently, Brandt and Santa-Clara (2006). In contrast to these seminal contributions, our specification of the strategic asset allocation problem is tractable and provides a simple econometric solution to the portfolio problem over long horizons without having to rely on dynamic stochastic programming as in Brennan et al. (1997), parametric specifications of the joint dynamics of the state variables and the portfolio returns as in Campbell et al. (2003) and Brandt and Santa-Clara (2006) or on complex methods to expand the asset space with artificially managed portfolios as in Brandt and Santa-Clara (2006). Our modeling and estimation strategy is also related to Britten-Jones (1999). This author derives the optimal portfolio weights for a myopic, mean-variance investor, as the coefficients of an OLS regression. In our case we obtain the model parameter estimates by GMM applied to a multiperiod Euler equation.

The rest of the article is structured as follows. Section 2 introduces the investor's maximization problem over a multiperiod horizon and discusses the solution to the problem for constant relative risk averse investors. Section 3 discusses suitable econometric methodology to estimate the optimal portfolio weights and make inference on these parameters. The section also presents some additional specification tests to assess the suitability of the proposed linear portfolio policy for describing the variation of the optimal portfolio weights and, more specifically, the choice of state variables for forecasting changes in the investment opportunity set. Section 4 presents an empirical application that compares the investment strategy of a myopic investor maximizing one period utility against the investment strategy of an investor maximizing a multiperiod objective function. Section 5 concludes.

2 The Model

Consider the portfolio choice of an investor who maximizes the expected utility of wealth (w_t), defined in real terms, over K periods. Assume that the multiperiod utility function is additively time separable and exhibits constant relative risk aversion. Then, an investor maximizes

$$\sum_{j=0}^{K} \beta^j E_t \left[\frac{w_{t+j}^{1-\gamma}}{1-\gamma} \right], \tag{1}$$

with $\gamma > 0$ and $\gamma \neq 1$. Two parameters describe CRRA preferences. The discount factor β measures patience, the willingness to give up wealth today for wealth tomorrow. The coefficient γ captures risk aversion, the reluctance to trade wealth for a fair gamble over wealth today.

The standard multiperiod maximization problem is completed by defining the accumulation equation determining the buildup of real wealth by the investor over time:

$$w_{t+1} = (1 + r_{t+1}^p) w_t. \tag{2}$$

The investor begins life with an exogenous endowment $w_0 \geq 0$. At the beginning of the period $t+1$ the investor receives income from allocating the wealth accumulated at time t in an investment portfolio offering a real return r_{t+1}^p. The return on this portfolio is defined as

$$r_{t+1}^p(\alpha_t) = r_{f,t+1} + \alpha_t' r_{t+1}^e, \tag{3}$$

with $r_{t+1}^e = (r_{1,t+1} - r_{f,t+1}, \ldots, r_{m,t+1} - r_{f,t+1})'$ denoting the vector of excess returns on the m risky assets over the real risk-free rate $r_{f,t+1}$, and $\alpha_t = (\alpha_{1,t}, \ldots, \alpha_{m,t})'$.

2.1 Optimal portfolio choice under constant relative risk aversion

In this section we derive the first order conditions of the optimal portfolio choice for a risk-averse investor exhibiting a power utility function with $\gamma > 1$. These conditions are obtained for a dynamic portfolio with optimal weights determined by a linear parametric specification in terms of a vector of state variables. We note first that investor's wealth at time $t+j$ can be expressed as

$$w_{t+j} = \prod_{i=1}^{j} (1 + r^{p}_{t+i}(\alpha_{t+i-1}))w_t. \tag{4}$$

Hence, the investor's objective function (1) becomes

$$\max_{\{\alpha_{t+j}\}} \left\{ \sum_{j=1}^{K} \beta^j \frac{w_t^{1-\gamma}}{1-\gamma} E_t \left[\prod_{i=1}^{j} (1 + r^{p}_{t+j+1-i}(\alpha_{t+j-i}))^{1-\gamma} \right] \right\}.$$

In order to be able to solve a multiperiod maximization problem that accommodates in a parsimonious way arbitrarily long investment horizons we entertain the parametric portfolio policy rule introduced in the seminal contributions of Aït -Sahalia and Brandt (2001), Brandt and Santa-Clara (2006) and Brandt et al. (2009):

$$\alpha_{h,t+i} = \lambda_h' z_{t+i}, \;\; h = 1, \ldots, m, \tag{5}$$

with $z_t = (1, z_{1,t}, \ldots, z_{n,t})'$ a set of state variables describing the evolution of the economy and reflecting all the relevant information available to the individual at time t, and $\lambda_h = (\lambda_{h,1}, \ldots, \lambda_{h,n})'$ the corresponding vector of parameters. Time variation of the optimal asset allocation is introduced through the dynamics of the state variables. This specification of the portfolio weights has two main features. First, it allows us to study the marginal effects of the state variables on the optimal portfolio weights through the set of parameters λ, and second, it avoids the introduction of time consuming dynamic stochastic programming methods.

A potential downside of this parametric approach is to force the individual's optimal portfolio policy rule to be linear and with the same parameter values over the long term horizon. Nevertheless, for finite horizon ($K < \infty$) objective functions, more sophisticated models can be developed that entertain different parametric portfolio policy rules for different investment horizons $i = 1, \dots, K$. This approach significantly increases the computational complexity of the methodology and is left for future research.

Under the above parametrization of the portfolio weights the maximization problem (1) becomes

$$\max_{\{\lambda_{hs}\}} \left\{ \sum_{j=1}^{K} \beta^j \frac{w_t^{1-\gamma}}{1-\gamma} E_t \left[\prod_{i=1}^{j} (1 + r_{t+i}^p(\lambda_h' z_{t+i-1}))^{1-\gamma} \right] \right\}. \tag{6}$$

The first order conditions of this optimization problem with respect to the vector of parameters λ_{hs}, with $h = 1, \dots, m$ and $s = 1, \dots, n$, provide a system of mn equations characterized by the following conditions:

$$E_t \left[\sum_{j=1}^{K} \beta^j \psi_{t,j}(z_s; \lambda_h) \right] = 0, \tag{7}$$

with

$$\psi_{t,j}(z_s; \lambda_h) = \left(\sum_{i=1}^{j} \frac{z_{s,t+i-1} r_{h,t+i}^e}{1 + r_{t+i}^p(\lambda_h' z_{t+i-1})} \right) \left(\prod_{i=1}^{j} (1 + r_{t+i}^p(\lambda_h' z_{t+i-1}))^{1-\gamma} \right). \tag{8}$$

The introduction of the vector of state variables z_t allows us to incorporate forecasts of the investment opportunity set in the optimal allocation of assets. The system of equations makes allowance for an infinite horizon model given by considering $K = \infty$ in the above expression. The moment condition (7) introduces additional restrictions to the system leading to an overidentified model characterized by the vector of unconditional moment conditions

$$E \left[\sum_{j=1}^{K} \beta^j \psi_{t,j}(z_s; \lambda_h) \otimes z_t \right] = 0 \tag{9}$$

where $\otimes$ denotes element by element multiplication. This system of equations provides a simple alternative to the setting introduced by Brandt and Santa-Clara (2006) to derive the optimal portfolio weights in dynamic settings. In contrast to this seminal contribution our approach does not require expanding the asset space with managed and timing portfolios. The algebra is also considerably simpler.

As mentioned earlier, our approach can also accommodate alternative formulations of the portfolio policy rule beyond the linear specification (5). However, for simplicity and consistency with the related literature we consider the linear portfolio policy. Our model, nevertheless, provides the flexibility to naturally develop specification tests to assess the validity of the parametric portfolio policy rule and the choice of the state variables. This is carried out in the next section.

To complete the section we also discuss the truncation of the infinite horizon model characterizing the investor's strategic behavior. To do this we note that as j approaches K, the contribution of the functions $\psi_{t,j}(z_s; \lambda_h)$ in (7) converges to zero justifying a truncation of the infinite horizon model. We propose a truncation determined by K^* with $K^* = \min\{j \mid \beta^j \leq tol,\ j = 1, \ldots, K\}$ with tol a tolerance level at the discretion of the researcher. To facilitate the choice of a universal tolerance level independent of the magnitude of the weights, condition (7) is rescaled such that the sum of the K weights is equal to 1. Under this condition each equation of the system defined in (7) can be written as a convex combination of the functions $\psi_{t,j}(z_s; \lambda_h)$. More specifically, the optimal portfolio problem is the solution to

$$E\left[\sum_{j=1}^{K} w^*_{j,K} \psi_{t,j}(z_s; \lambda_h) \otimes z_t\right] = 0, \tag{10}$$

with $w^*_{j,K} = \frac{1-\beta}{1-\beta^{K+1}}\beta^j$. This representation of the system of equations is particularly relevant for the infinite horizon model for which $w^*_{j,\infty} = (1-\beta)\beta^j$ and the truncation of the infinite terms by a finite K^* satisfying the above condition is particularly desirable in practical applications.

3 Econometric methodology

This section discusses an application of the generalized method of moments that accommodates the convex combination of moment conditions derived in (10). This methodology leads to an overidentified system of equations that allows us not only to estimate and make statistical inference on the parameters determining the optimal portfolio policy but also to test for the correct specification of such policy and variations of it.

3.1 Estimation

A suitable empirical representation of the Euler equation (10) is

$$\widehat{\phi}_{h,s}(z_s;\lambda_h) = \frac{1}{T-K^*}\sum_{t=1}^{T-K^*}\left(\sum_{j=1}^{K^*} w^*_{j,K}\psi_{t,j}(z_s;\lambda_h)\otimes z_t\right) = 0, \tag{11}$$

where T is the sample size used for estimating the model parameters; $K^* = K$ for the finite multiperiod case and $K^* = \min\{j \mid w^*_{j,\infty} \leq tol,\ j = 1,\ldots,\infty\}$ for the infinite horizon case. For each pair (h,s), condition (11) yields a $n \times 1$ vector of moment conditions. Let

$$\widehat{\phi}^{(\widetilde{s})}_{h,s}(z_s;\lambda_h) = \frac{1}{T-K^*}\sum_{t=1}^{T-K^*}\left(\sum_{j=1}^{K^*} w^*_{j,K}\psi_{t,j}(z_s;\lambda_h)\ z_{\widetilde{s},t}\right) \tag{12}$$

denote each element of such vector with $\widetilde{s} = 1,\ldots,n$ where $z_{1,t} = 1$; and let $g_T(c)$ be the $mn^2 \times 1$ vector that stacks each of the sample moments $\widehat{\phi}^{(\widetilde{s})}_{h,s}$ indexed by h, s and $\widetilde{s}$, with $h = 1,\ldots,m$ and $s,\widetilde{s} = 1,\ldots,n$. The idea behind GMM is to choose $\widehat{\lambda}$ so as to make the sample moments $g_T(c)$ as close to zero as possible. More formally, the GMM estimator of the matrix of λ parameters is defined as

$$\widehat{\lambda}_T = \underset{c\in\Lambda}{\arg\min}\ g_T(c)'\widehat{V}_T^{-1}g_T(c)$$

where $\widehat{V}_T$ is an $mn^2 \times mn^2$, possibly random, non-negative definite weigth matrix, whose rank is greater than or equal to mn. This matrix admits different representations. In a first stage $\widehat{V}_T$ can be the identity matrix or some other matrix, as for example, $I_{mn} \otimes Z'Z$, with I_{mn} the identity matrix of dimension mn and Z the $(T-K) \times n$ matrix corresponding to the state variables. In a second stage, to gain efficiency, this matrix is replaced by a consistent estimator of the asymptotic covariance matrix V_0 of the random vector $g_T(c)$. A natural estimator of this matrix is discussed in (15).

Statistical inference for the portfolio weights is obtained by applying asymptotic theory results from GMM estimation to the quantity $g_T(c)$. To show this we assume the portfolio returns and state variables to be jointly stationary, and derive the consistency and asymptotic normality of $\widehat{\phi}_{h,s}^{(\widetilde{s})}$. In particular, we note that for K^* fixed, condition

$$\sum_{t=-\infty}^{\infty} \sum_{j=1}^{K^*} \sum_{j'=1}^{K^*} w_{j,K}^* w_{j',K}^* E \left| \psi_{t,j}(z_s;\lambda_h) \psi_{t',j'}(z_s;\lambda_h) \; z_{\widetilde{s},t} z_{\widetilde{s},t'} \right| < \infty \tag{13}$$

is sufficient to guarantee that

$$\widehat{\phi}_{h,s}^{(\widetilde{s})} \xrightarrow{p} \phi_{h,s}^{(\widetilde{s})} = E \left[\sum_{j=1}^{K^*} w_{j,K}^* \psi_{t,j}(z_s;\lambda_h) \; z_{\widetilde{s},t} \right], \tag{14}$$

with $\xrightarrow{p}$ denoting convergence in probability, as the sample size T increases. This condition depends on the persistence of the portfolio returns, and in particular, of the set of state variables $z_{\widetilde{s},t}$. Similarly, under suitable regularity conditions, the central limit theorem implies that

$$\sqrt{T} \left(\widehat{\phi}_{h,s}^{(\widetilde{s})} - \phi_{h,s}^{(\widetilde{s})} \right) \xrightarrow{d} N \left(0, V_{hs}^{(\widetilde{s})} \right)$$

with $N \left(0, V_{hs}^{(\widetilde{s})} \right)$ denoting a Normal distribution with $V_{hs}^{(\widetilde{s})}$ the relevant asymptotic variance.

Following similar arguments for the different elements of the vector $g_T(c)$ it is not difficult to see that under condition (10) the standardized vector $g_T(c)$ converges in distribution to a multivariate Normal distribution with covariance matrix $V_0 = E\left[g_T(c)g_T'(c)\right]$ defined by the elements $E\left[\phi_{h_1,s_1}^{(\widetilde{s}_1)}\phi_{h_2,s_2}^{(\widetilde{s}_2)}\right]$ with $s_1, \widetilde{s}_1, s_2, \widetilde{s}_2 = 1, \ldots, n$ and $h_1, h_2 = 1, \ldots, m$. This matrix can be consistently estimated by its sample counterpart $\widehat{V}_T$ defined by the elements

$$\frac{1}{(T-K^*)^2}\sum_{t=1}^{T-K^*}\sum_{j=1}^{K^*}\sum_{j'=1}^{K^*} w_{j,K}^* w_{j',K}^* \psi_{t,j}(z_{s_1}, \lambda_{h_1})\psi_{t,j'}(z_{s_2}, \lambda_{h_2})\ z_{\widetilde{s}_1,t} z_{\widetilde{s}_2,t}$$

$$+\frac{1}{(T-K^*)^2}\sum_{t=1}^{T-K^*}\sum_{\substack{t'=1 \\ t'\neq t}}^{T-K^*}\sum_{j=1}^{K^*}\sum_{j'=1}^{K^*} w_{j,K}^* w_{j',K}^* \psi_{t,j}(z_{s_1}, \lambda_{h_1})\psi_{t',j'}(z_{s_2}, \lambda_{h_2})\ z_{\widetilde{s}_1,t} z_{\widetilde{s}_2,t'} \tag{15}$$

with $s_1, \widetilde{s}_1, s_2, \widetilde{s}_2 = 1, \ldots, n$ and $h_1, h_2 = 1, \ldots, m$. This estimator highlights the strong persistence in the covariance matrix V_0. This persistence is due to the overlapping of periods produced by considering an strategic investment horizon ($K^* > 1$) in the investor's objective function.

The above properties also establish the asymptotic distribution of the parameter estimators defining the optimal portfolio weights, that satisfy

$$\sqrt{T}\left(\widehat{\lambda}_T - \lambda\right) \xrightarrow{d} N(0, W_0) \tag{16}$$

with $W_0 = (D_0\Omega_0^{-1}D_0)^{-1}$, where $\Omega = E\left[g(c)g'(c)\right]$, $D_0 \equiv D(c) = \frac{\partial g(c)}{\partial c}$ and $D(c)$ is continuous at $c = \lambda$. The vector $g(c)$ stacks each of the $mn^2 \times 1$ population moments $\phi_{h,s}^{(\widetilde{s})}$ defined in (14).

3.2 Specification tests

This section discusses three statistical tests based on the overidentified system of equations introduced above. We discuss first an specification test to assess the validity of the linear policy rule (5). The second test is an incremental Sargan type test concerned with testing the relevance of the state variables within the linear specification. The third test allows us to compare different specifications of the investor's maximization problem in terms of the investment horizon. A particularly interesting application of the latter is to compare the myopic and strategic asset allocations.

The systems of equations defined in (10) entail the existence of testable restrictions implied by the econometric model. Estimation of λ sets to zero mn linear combinations of the mn^2 sample orthogonality conditions $g_T(c)$. The correct specification of the model implies that there are $mn(n-1)$ linearly independent combinations of $g_T(\widehat{\lambda}_T)$ that should be close to zero but are not exactly equal to zero. This hypothesis is tested using the Hansen test statistic (Hansen, 1982). Let $s(\widehat{\lambda}_T) = g_T(\widehat{\lambda}_T)'\widehat{V}_T^{-1}g_T(\widehat{\lambda}_T)$ with $\widehat{V}_T$ defined in (15), that under the null hypothesis of correct specification of the model, satisfies

$$s(\widehat{\lambda}_T) \xrightarrow{d} \chi^2_{mn(n-1)}. \tag{17}$$

In contrast to the standard GMM specification tests for independent and identically distributed (*iid*) observations, in our framework the components of the vector $g_T(\widehat{\lambda}_T)$ exhibit strong serial correlation due to the overlapping of investment horizons. Hence, it is not advisable to assume that $V_0 = \Omega_0/(T - K^*)$ as it would be the case in the *iid* case. Therefore, an appropriate standardization to achieve a chi-square distribution under the null hypothesis is obtained by the construction of $s(\widehat{\lambda}_T)$ presented above and not by $(T-K^*)g_T(\widehat{\lambda}_T)'\widehat{\Omega}_T^{-1}g_T(\widehat{\lambda}_T)$, with $\widehat{\Omega}_T$ a consistent estimator of Ω_0 defined as

$$\widehat{\Omega}_T = \frac{1}{T-K^*}\sum_{t=1}^{T-K^*}\sum_{j=1}^{K^*}\sum_{j'=1}^{K^*} w^*_{j,K}w^*_{j',K}\psi_{t,j}(z_{s_1},\lambda_{h_1})\psi_{t,j'}(z_{s_2},\lambda_{h_2})\ z_{\widetilde{s}_1,t}z_{\widetilde{s}_2,t}.$$

The second test, based on the incremental Sargan (1958, 1959) tests, assesses the appropriateness of different subsets of the moment conditions. In particular, we apply it to test the relevance of the state variables within the linear specification (5). Consider a set of $n-2$ state variables defining a vector z_t of dimension $n-1$, obtained from also including a constant in the portfolio policy specification, and take

$$E_t\left[\sum_{j=1}^{K} w_{j,K}^{*}\psi_{t,j}(z_s;\lambda_h)\right]=0 \tag{18}$$

for $h=1,\ldots,m$ and $s=1,\ldots,n-1$ as the maintained hypothesis. We wish to test the correct specification of the linear policy rule (5) obtained from including an additional state variable $z_{n,t}$ to the vector z_t. To do this we need to test the suitability of the above set of restrictions for $s=n$ and $h=1,\ldots,m$. This implies a joint test of $(2n-1)m$ conditions obtained as the difference between mn^2 and $m(n-1)^2$ restrictions. The incremental Sargan test is defined in this context as

$$S_d = s(\widehat{\lambda}_T) - s_1(\widehat{\lambda}_{1T}) \tag{19}$$

with $s_1(\widehat{\lambda}_{1T}) = g_{1T}(\widehat{\lambda}_{1T})'\widehat{V}_{1T}^{-1}g_{1T}(\widehat{\lambda}_{1T})$, where $g_{1T}(\widehat{\lambda}_{1T})$ denotes the subset of $g_T(\widehat{\lambda}_T)$ determined by $m(n-1)^2$ conditions in (11), $\widehat{\lambda}_{1T}$ is the vector minimizing $s_1(\widehat{\lambda}_{1T})$ and $\widehat{V}_{1T}$ is the $m(n-1)\times m(n-1)$ version of $\widehat{V}_T$ defined in (15). This test statistic converges under the null hypothesis given by the correct specification of $z_{n,t}$ in the linear portfolio rule (5) to a chi-square distribution with $(2n-1)m$ degrees of freedom.

The third test is a Hausman type test that allows us to compare different specifications of the investor's maximization problem in terms of the investment horizon. More specifically, we contemplate a short-term and a long-term investment horizon and compare the informational content of the period spanning between the short and long-term horizons. Under the null hypothesis, the informational content of this period is null implying that the relevant terms in (7) are equal to zero. For K finite, this test can be interpreted as a Hausman type test because whereas under the null hypothesis the parameter estimates corresponding to the short-term investment horizon are consistent they are not under the alternative hypothesis.

Let $K_1 < K_2$ denote the short-term and long-term horizons, and consider the joint null hypothesis

$$H_0 : E\left[\sum_{j=K_1+1}^{K_2} w^*_{j,K_2}\psi_{t,j}(z_s;\lambda_h) \otimes z_t\right] = 0, \tag{20}$$

for $h = 1,\ldots,m$ and $s = 1,\ldots,n$. The consistency of the sample moments entails under the null hypothesis that

$$\frac{1}{T-K^*}\sum_{t=1}^{T-K^*}\left(\sum_{j=K_1+1}^{K_2} w^*_{j,K_2}\psi_{t,j}(z_s;\lambda_h) z_{\widetilde{s},t}\right) \xrightarrow{p} 0 \tag{21}$$

for all $h = 1,\ldots,m$ and $s,\widetilde{s} = 1,\ldots,n$. A suitable test statistic for this joint hypothesis is $s_r(\widehat{\lambda}^o_T)$ with $s_r(\widehat{\lambda}^o_T) = g_{rT}(\widehat{\lambda}^o_T)'\widehat{V}_T^{-1}g_{rT}(\widehat{\lambda}^o_T)$, where $g_{rT}(c)$ is the vector of moment conditions stacking the mn^2 sample moments in (21) and $\widehat{\lambda}^o_T$ denoting the set of parameters estimated under the null hypothesis (20). Under the null hypothesis H_0, the test statistic satisfies

$$s_r(\widehat{\lambda}^o_T) \xrightarrow{d} \chi^2_{mn^2}. \tag{22}$$

A natural application of this test is to compare the suitability of the myopic asset allocation problem given by $K_1 = 1$ against the strategic allocation given by some K_2 determining the long-term horizon. Under the null hypothesis both maximization problems should yield similar optimal portfolio weights and entail, hence, the consistency of the λ parameter estimates in both scenarios. The alternative hypothesis implies differences in the optimal portfolio allocation between the myopic and strategic asset allocations. This test can also be applied to assess the suitability of truncations of the infinite horizon model determined by K^*. To do this we consider $K_1 = K^*$ and $K_2 = K^* + \epsilon$ with $\epsilon > 0$ some large arbitrary number. Under the null hypothesis

(20), the truncation of the infinite horizon model given by K^* provides consistent estimates of the optimal portfolio weights.

4 Empirical application

In this section we analyze the one-month horizon investor strategy intended to represent the myopic investor (Brennan et al., 1997) versus the infinite time horizon strategy that is intended to reflect the strategic investor. Following similar studies such as Brennan et al. (1997), Brandt (1999) and Campbell et al. (2003), we consider an investor that can allocate wealth among stocks, bonds and the one-month real Treasury bill rate. By doing this we implicitly take into account the role played by inflation in the formation of optimal portfolios. As in Campbell et al. (2003), we do not impose short-selling restrictions.

The time-variation of the investment opportunity set is described by a set of state variables that have been identified in the empirical literature as potential predictors of the excess stock and bond returns and the short-term ex-post real interest rates. These variables are the detrended short-term interest rate (Campbell, 1991), the U.S. credit spread (Fama and French, 1989), the S&P 500 trend (Aït-Sahalia and Brandt, 2001) and the one-month average of the excess stock and bond returns[1] (Campbell et al., 2003). The detrended short-term interest rate detrends the short-term rate by subtracting a 12-month backwards moving average. The U.S. credit spread is defined as the yield difference between Moody's Baa- and Aaa-rated corporate bonds. The S&P 500 momentum is the difference between the log of the current S&P 500 index

[1]Unreported results including the dividend yield in our set of state variables confirm the absence of statistical significance of the variable. This empirical finding is consistent with studies such as Lettau and Ludvigson (2001) and Goyal and Welch (2003) that point out that predictability by the dividend yield is not robust to the addition of the 1990's decade. More empirical evidence supporting the omission of the dividend yield from our set of state variables is provided by Ang and Bekaert (2007). These authors note that at long horizons, excess return predictability by the dividend yield is not statistically significant, not robust across countries, and not robust across different sample periods. In this sense the predictability that has been the focus of most recent finance research is simply not there.

level and the average index level over the previous 12 months. We demean and standardize all the state variables in the optimization process (Brandt et al, 2009).

4.1 Data description

Our data covers the period January 1980 to December 2010. We collect monthly data from Bloomberg on the S&P 500 and G0Q0 Bond Index. The G0Q0 Bond Index is a Bank of America and Merrill Lynch U.S. Treasury Index that tracks the performance of U.S. dollar denominated sovereign debt publicly issued by the U.S. government in its domestic market. We collect the nominal yield on the U.S. one-month risk-free rate from the Fama and French database. Finally, we collect the consumer price index (CPI) time series and the yield of the Moody's Baa- and Aaa-rated corporate bonds from the U.S. Federal Reserve.

Table 1 reports the sample statistics of the annualized excess stock return, excess bond return and short-term ex-post real interest rates. The bond market outperforms the stock market during this period. In particular, the excess return on the bond index is higher than for the S&P 500 and exhibits a lower volatility entailing a Sharpe ratio almost three times higher for bonds than stocks. Additionally, the excess bond return has larger skewness and lower kurtosis. This anomalous outperformance of the G0Q0 index versus the S&P 500 is mainly explained by the last part of the sample and the consequences of the subprime crisis on the valuation of the different risky assets.

[Insert Tables 1, 2 and 3 about here]

Table 2 shows the estimates of the seemingly unrelated regression estimation of the excess stock return, excess bond return and short-term ex-post real interest rate using as explanatory variables the detrended short-term interest rate, the U.S. credit spread, the S&P 500 trend and the one-month average of the excess stock and bond returns. Table 3 reports the correlation of the state variables and the excess stock return, excess bond return and short-term ex-post real interest rate innovations obtained from the SURE model. These estimates allow us to obtain some insights on the dynamics of excess stock and bond returns and their variation over time linked to the state variables that we assume are driving the change in the investment opportunity set. A first conclusion that can be drawn from the estimated model, and in particular from the low R^2 statistics reported, is the difficulty in predicting excess asset returns.

4.2 Strategic vs. myopic asset allocation

We study the optimal portfolio problem of an investor that faces a time-varying investment opportunity set that can be forecast by the set of state variables discussed above. We distinguish between the short-term investor, whose time horizon is one-month and is intended to represent the myopic strategy (Brennan et al., 1997), and a long-term investor that is infinitely long lived. The differences in the optimal portfolios between the short-term and long-term investor are explained by the role of hedging that the different assets offer against changes in the time-varying investment opportunity set (Merton, 1973; Brennan et al., 1997; Campbell and al., 2003).

We compute optimal portfolio rules for $\gamma = 2, 5, 20$ and 100, and assume a value of $\beta = 0.95$. We provide an approximate solution to the infinitely long lived investor by choosing a value of $K^* = 100$ in the system of equations given in (11). This restriction implies a tolerance level of 0.03 determined by the standardized weight function $w^*_{j,\infty} = \beta^j(1-\beta)$ obtained from assuming that the individual invests all of its wealth in the financial portfolio. A two-step Gauss-Newton type algorithm using numerical derivatives is implemented to estimate the model parameters. In a first stage we initialize the covariance matrix $\widehat{V}_T$ with the matrix $I_{mn} \otimes Z'Z$, and in a second stage, after obtaining a first set of parameter estimates, we repeat the estimation replacing this matrix by (15). This matrix $\widehat{V}_T$ is also used to perform the different specification tests described below.

Our theoretical framework has several advantages over other methods proposed in the literature for solving the strategic asset allocation problem. First, under the assumption that the optimal portfolio policy rule depends on the realization of the state variables we can directly estimate the optimal strategic portfolio allocation to stocks, bonds and cash without estimating asset expected returns, volatilities and correlations. Second, we take advantage of the overidentified system of equations to statistically test the correct specification of (5) and provide statistical evidence on the validity of the state variables. Third, the tests presented in the preceding section also allow us to statistically confirm the differences between the myopic and strategic asset allocation problems.

Table 4 reports the estimates of the optimal myopic and strategic asset allocation rules specified in (5). We assume that both portfolio allocations to stocks and bonds are linearly related to our set of state variables, and therefore, the fraction of wealth allocated to stocks, bonds and the one-month Treasury bill changes over time. Table 4 reports parameter estimates, t-statistics and the test statistic (17) corresponding to the specification test given by the overidentified system of equations. The one-month average of the excess stock and bond returns is the only variable that is significantly and positively related to the myopic allocation to the S&P 500, while the myopic allocation to bonds is found to be negatively and significantly related to the average of one-month excess stock and bond returns, the U.S. credit spread and the S&P 500 trend. The sign pattern is consistent across different risk aversion parameters γ with increasing estimates of the beta coefficients associated with decreasing values of γ revealing the existence of an inverse relationship between the degree of investor's risk aversion and its responsiveness to changes in the information set. Figure 1 plots the optimal myopic allocation to stocks and bonds for an investor with $\gamma = 5$ and shows that the optimal asset allocation responds aggressively to changes in the information set as proxied by the set of state variables.

[Insert Table 4 about here]

Our econometric specification shows that the strategic allocation to the S&P 500 and bond indices differs in two main aspects with respect to the myopic asset allocation. First, the absolute value of the coefficients of the significant variables in the strategic case is usually larger than in the myopic case, and more importantly, these parameters are estimated more precisely as shown by the lower p-values. Second, the strategic allocation to the S&P 500 is found to be positively and significantly related to the trend variable and negatively to the detrended short-term interest rate. The relevance of the detrended short-term interest rate is also noted from the strategic allocation to bonds. In this case this state variable is found to be negatively and significantly related to the dynamic optimal allocation to bonds for investors characterized by higher values of the risk aversion coefficient. These results clearly suggest that the investment horizon affects the relationship between the optimal allocation to assets in the portfolio and the state variables. Figure 2 plots the optimal stocks and bonds' hedging demand for an investor with $\gamma = 5$. There are some remarkable aspects to be noted from the graph. For example, the hedging demand varies significantly with the state variables rather than being stable along time. The sign of the hedging demand can also change depending on the realization of the state variables implying that the strategic asset allocation can be larger or lower than the myopic one.

[Insert Figures 1 and 2 about here]

To gain better understanding of the differences between the myopic and strategic asset allocations we compute the mean myopic allocation to each asset as well as the mean hedging portfolio demand for different values of γ. The sum of the optimal myopic and hedging components equals the optimal strategic allocation to each asset, see Campbell et al. (2003) for a discussion of this result. Table 5 reports how the myopic and hedging demands vary with the degree of risk aversion. The optimal myopic demand consists of a long position in stocks and bonds, and a short position in the one-month Treasury bill (cash). As expected, the mean optimal myopic portfolio allocation is tilted towards bonds, which have the largest Sharpe ratio among the three asset classes considered in our sample, entailing an optimal ratio of bonds to stocks of about 2.4. The results reported in Table 5 also suggest that the optimal allocation to risky assets, such as stocks and bonds, decreases as the relative risk aversion coefficient takes on larger values making the optimal myopic allocation to cash increase as investors become more conservative.

[Insert Table 5 about here]

The intertemporal hedging demand for bonds is positive reaching a percentage of 15% of the total strategic asset allocation and implying that the optimal mean strategic allocation to bonds is larger than the optimal mean myopic allocation to bonds. Our simple SURE model reports a negative relationship between the average of the one-month excess stock and bond return shocks and expected excess bond returns. The model also uncovers the existence of a positive correlation between the unexpected excess bond return and the average of one-month excess stock and bond return shocks. This finding suggests that poor bond returns are correlated with improvements in the future investment opportunity set. As a result, we note that bonds can be used to hedge time variation in their own returns making the strategic investor allocate a higher fraction of wealth to bonds. This effect is more important for more aggressive investors that are especially exposed to bond market fluctuations. This reasoning could also rationalize the low absolute value of the coefficient linked to the average stock and excess bond returns for the optimal strategic asset allocation to bonds compared to the absolute value of the coefficient in the myopic case.

Table 5 also shows that the mean intertemporal hedging demand for stocks is slightly negative and not very large in absolute terms. This result constitutes a difference with recent literature that finds a very positive intertemporal hedging demand for stocks, especially linked to the dividend yield variable (Brennan et al., 1997; Campbell et al., 2003). A possible explanation of this empirical finding could be based on the correlation structure of the state variables and the relationship of these state variables with the expected stock excess return. Since the one-month average of the excess stock and bond returns is positively related to the expected stock excess return, and the S&P 500 trend and the detrended short-term interest rate are negatively related to the expected stock excess return, poor stock returns coincident with negative shocks to the one-month average of the excess stock and bond returns and the S&P 500 trend variable have opposing effects on the future investment opportunity set. On the one hand, the one-month average of the excess stock and bond returns and the detrended short-term interest rate predict a deterioration of the investment opportunity set by reducing the expected stock excess return and making the intertemporal hedging demand for stocks decrease. On the other hand, the S&P 500 trend anticipates an improvement of the investment opportunity set and makes the intertemporal hedging demand for stocks increase. These opposing effects almost cancel each other out.

4.3 Specification tests

The test of overidentifying restrictions in (17) shows that all the strategic asset allocation models estimated under different assumptions on the degree of risk aversion are well specified (p-values larger than 0.2) with the only exception of the model that considers $\gamma = 2$ that reports a p-value of 0. To provide further statistical evidence on the validity of our specification of the policy rule and the state variables we also carry out the tests based on the incremental Sargan test discussed above. To do this we concentrate on the case $\gamma = 5$, $\beta = 0.95$ and $K^* = 100$, and analyze the statistical significance of the state variables comprising our linear policy rule.

We analyze first the linear portfolio policy exclusively considering the detrended short-term interest rate (Tb). Note that in this case expression (5) becomes $\alpha_{h,t+j} = \lambda_{h,1} + \lambda_{h,2} Tb_{t+j-1}$, implying a value of $n = 2$ and $m = 2$ risky assets as parameters in the incremental Sargan test. The test statistic takes a value of 1.83 and a p-value of 0.93 obtained from a chi-square distribution with 6 degrees of freedom. This result establishes the statistical relevance of the specification, and in particular of the state variable, to solve the strategic investor asset allocation problem. We add to this specification the one month average of the excess stock and bond return. The test statistic takes a value of 12.30 obtained as the difference between 14.13 and 1.83. The p-value, obtained from a chi-square distribution with 10 degrees of freedom, is 0.26 validating the augmented specification of (5). Similarly, for the U.S. credit spread, we obtain a test statistic of 11.15 obtained as the difference between 25.28 and 14.13 yielding a p-value of 0.67 obtained from a chi-square distribution with 14 degrees of freedom. Finally, for the S&P 500 trend we observe a test statistic of 14.23 obtained as the difference between 39.50 and 25.28 yielding a p-value of 0.71 obtained from a chi-square distribution with 18 degrees of freedom. These results shed sufficient statistical evidence to accept the marginal relevance of each of the variables comprising our policy rule.

We also test statistically the suitability of the truncation of the infinite horizon model given by different values of the truncation parameter K^* in (11) for the same parametrization of the investor's maximization problem ($\gamma = 5$ and $\beta = 0.95$). In particular, we have considered $K^* = 10, 18, 32, 60, 70, 80, 90, 95$ and 100. To do this, we compute the test (22) using these different values of K^* as K_1 and considering $K_2 = 110$; as a robustness exercise, we have repeated the tests also considering $K_2 = 150$. The null hypothesis implies that horizons further than K_1 periods ahead do not contain relevant information to the investor. This condition is equivalent to ascertaining that the investor's multiperiod maximization problem is really finite. The tests corresponding to different values of K_1 reveal that the null hypothesis (20) is only accepted for values of K^* around 100 and higher. In particular, the Wald type test reports a statistic of 1.11 for $K_2 = 110$ and 0.41 for $K_2 = 150$, respectively. These results provide sufficient statistical evidence to accept the choice of $K^* = 100$ as a valid approximation of the infinite horizon problem.

Finally, we implement a version of this test that statistically compares the myopic and strategic asset allocation problems. More precisely, we check whether the optimal solution characterized by $K = 1$ in (11) reports the same optimal portfolio weights as the strategic allocation characterized by $K = 100$ that proxies the infinite horizon problem. To do this, we simply consider $K_1 = 1$ and $K_2 = 100$ in the hypothesis test (20). The test statistic reports a value of 5.98 providing sufficient evidence to reject statistically the hypothesis that the portfolio weights obtained from these portfolio allocations are equal.

Other avenues have been recently explored to compare the performance of dynamic and myopic portfolios beyond the differences in optimal portfolio weights and portfolio returns discussed in this paper. Thus, Lan (2014) computes the utility cost associated with adopting a myopic rather than a dynamic strategy. Interestingly, this author finds that when dynamic policies are implemented in a real-time, out-of-sample setting, an investor may actually be better off relying on myopic, rather than dynamic policies. This analysis consists of comparing the nonlinear transformations of the optimal portfolio returns provided by the utility functions defining each approach. This analysis is however beyond the scope of this paper and left for future research on the topic.

5 Conclusion

This paper has proposed a simple framework to study the investor's optimal asset allocation problem over long, potentially infinite, horizons. The method accommodates an arbitrarily large number of assets in the portfolio and state variables in the information set. In contrast to most of the literature on dynamic asset allocation the method does not rely on dynamic stochastic programming or numerical approximations. It is made operational through the first order conditions of the maximization problem of an strategic investor under the assumption that the optimal portfolio weights are described by a parametric linear portfolio policy.

We have applied standard GMM methodology to estimate the parameters driving the dynamics of the optimal portfolio weights and make inference. In particular, to the best of our knowledge, this is the first paper to propose a testing framework for the parametric portfolio policy specification in terms of its functional form and the composition of the state variables describing the information set. The empirical results for a portfolio of assets, bonds and the one-month real Treasury bill provide ample support to the suitability of a linear specification of the optimal portfolio weights determined by the detrended short-term interest rate, the U.S. credit spread, the S&P 500 trend and the one-month average of the excess stock and bond returns for investors exhibiting constant relative risk aversion.

Our results are also supportive of statistically significant differences across the myopic and strategic portfolio. More specifically, our application reveals that the absolute value of the optimal portfolio weights in the strategic case is usually larger than in the myopic case reflecting more aggressive responses to changes in the state variables in the strategic case. The analysis of the intertemporal hedging demand corresponding to the strategic allocation also reveals staggering differences with the myopic case. Thus, in constrast to recent literature that finds a very positive intertemporal hedging demand, we observe that the hedging demand for stocks is slightly negative and not very large in absolute terms. In our framework this result is due to the forecast ability of the average of the one-month excess stock and bond returns and the detrended short-term interest rate that predict a deterioration of the expected stock excess return. This negative effect is partly compensated by the forecast provided by the S&P 500 trend that anticipates an improvement of the investment opportunity set.

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TABLES AND FIGURES

Table 1: Sample statistics

	Mean	Volatility	Sharpe ratio	Skewness	Kurtosis
$R^e_{S\&P500}$	0.0266	0.131	0.21	-1.12	4.88
R^e_{Bonds}	0.0290	0.0566	0.51	0.15	2.17
r_f	0.0183	0.021		0.38	3.16

This table presents the sample statistics of the excess stock return $\left(R^e_{S\&P500}\right)$, excess bond return $\left(R^e_{Bonds}\right)$ and short-term ex-post real interest rates (r_f). The sample data covers the period January 1980 to December 2010. The return horizon is one month. Mean and volatility are expressed in annualized terms.

Table 2: Seemingly unrelated regression estimation of the excess stock return, excess bond return and short-term ex-post real interest rates.

$R^e_{S\&P500}$	T_b	Def	Trend	ARP	R^2
$\beta_{R^e_{S\&P500}}$	-0.23	-0.13	-0.02	1.19	0.10
p-value	0.25	0.51	0.90	0.00	
R^e_{Bonds}	T_b	Def	Trend	ARP	R^2
β_{Bonds}	-0.02	-0.04	-0.01	-0.02	0.03
p-value	0.82	0.62	0.15	0.01	
r_f	T_b	Def	Trend	ARP	R^2
β_{r_f}	-0.05	0.25	0.19	-0.08	0.06
p-value	0.33	0.00	0.00	0.11	

This table presents the estimates of the seemingly unrelated regression estimation (SURE) of the excess stock return $\left(R^e_{S\&P500}\right)$, excess bond return $\left(R^e_{Bonds}\right)$ and short-term ex-post real interest rates $\left(r_f\right)$, using as explanatory variables the detrended short-term interest rate (T_b), the U.S. credit spread (Def), the S&P 500 trend (Trend) and the one-month average of the excess stock and bond returns (ARP). We use monthly data from January 1980 to December 2010.

Table 3: Correlation matrix of the state variables with the excess stock return, excess bond return and short-term ex-post real interest rate innovations.

	$R^e_{S\&P500}$	R^e_{Bonds}	r_f	Tb	Def	Trend	ARP
$R^e_{S\&P500}$	1						
R^e_{Bonds}	-0.01	1					
r_f	-0.01	0.23	1				
Tb	-0.01	-0.14	-0.11	1			
Def	-0.05	0.07	0.07	-0.31	1		
Trend	0.21	0.00	0.00	0.21	-0.44	1	
ARP	0.87	0.40	0.14	-0.12	0.00	0.27	1

This table presents the estimated correlations of the state variables with the excess stock return $\left(R^e_{S\&P500}\right)$, excess bond return $\left(R^e_{Bonds}\right)$ and short-term ex-post real interest rate $\left(r_f\right)$ innovations obtained from the seemingly unrelated regression estimation (SURE). The state variables are the detrended short-term interest rate (T_b), the U.S. credit spread (Def), the S&P 500 trend (Trend) and the one-month average of the excess stock and bond returns (ARP). We use monthly data from January 1980 to December 2010.

Table 4: Myopic and Strategic asset allocation.

	Myopic CRRA=2	Strategic CRRA=2	Myopic CRRA=5	Strategic CRRA=5	Myopic CRRA=20	Strategic CRRA=20	Myopic CRRA=100	Strategic CRRA=100
S&P 500								
β_{Tb}	-1.10	-3.01	-0.40	-1.28	-0.09	-0.28	-0.01	-0.06
t-stat	(-1.21)	(-7.69)	(-1.11)	(-4.54)	(-1.04)	(-3.58)	(-0.69)	(-4.84)
β_{Def}	0.23	0.09	0.08	0.28	0.02	-0.08	0.01	-0.02
t-stat	(0.26)	(0.22)	(0.23)	(-0.97)	(0.26)	(-1.18)	(0.57)	(-1.18)
β_{Trend}	-0.63	0.84	-0.21	0.55	-0.04	0.13	-0.01	0.03
t-stat	(-0.65)	(2.91)	(-0.54)	(2.37)	(-0.45)	(1.81)	(-0.29)	(2.12)
β_{ARP}	6.09	8.37	2.34	3.71	0.56	0.90	0.11	0.15
t-stat	(5.55)	(23.71)	(5.33)	(13.35)	(5.22)	(9.08)	(4.78)	(10.67)
Bonds								
β_{Tb}	-1.13	0.19	-0.52	-0.48	-0.13	-0.20	-0.02	-0.04
t-stat	(-0.7)	(0.38)	(-0.8)	(-1.41)	(-0.85)	(-1.97)	(-0.88)	(-2.16)
β_{Def}	-5.99	-8.08	-2.45	-3.36	-0.63	-0.70	-0.14	-0.10
t-stat	(-3.43)	(-9.68)	(-3.39)	(-5.58)	(-3.45)	(-5.01)	(-3.62)	(-3.12)
β_{Trend}	-3.92	-7.00	1.61	-3.64	-0.40	-0.91	-0.07	-0.16
t-stat	(-2.01)	(-10.02)	(-2.04)	(-5.49)	(-2.01)	(-5.03)	(-1.73)	(-5.86)
β_{ARP}	-3.68	-1.46	-1.49	-1.05	-0.37	-0.33	-0.07	-0.04
t-stat	(-1.92)	(2.32)	(-1.89)	(-2.71)	(-1.86)	(-2.50)	(-1.72)	(-1.19)
χ^2		81.73		39.49		37.3		46.69
d.f		40		40		40		40
p-value		0.00		0.49		0.59		0.22

This table shows estimates of the optimal investment strategy policy of a myopic investor, whose time horizon is one month, and a strategic investor that is infinitely long lived. We consider an investor that can allocate her wealth among stocks, bonds and the one-month Treasury bill rate. The optimal portfolio rule is specified in equation (5) and optimized for a power utility function with different CRRA coefficients (γ=2, 5, 20, 100) and a value of δ=0.95, using these state variables: the detrended short-term interest rate (T_b), the U.S. credit spread (Def), the S&P 500 trend (Trend) and the one-month average of the excess stock and bond returns (ARP). We use monthly data from January 1980 to December 2010. The statistics are reported with *p*-values in parentheses and associated degrees of freedom (d.f.).

Table 5: Mean asset demands.

	Myopic CRRA=2	Hedging demand CRRA=2	Myopic CRRA=5	Hedging demand CRRA=5	Myopic CRRA=20	Hedging demand CRRA=20	Myopic CRRA=100	Hedging demand CRRA=100
S&P 500	2.53	0.27	1.05	-0.07	0.26	-0.04	0.05	0.00
Bonds	6.24	0.52	2.49	0.42	0.59	0.04	0.09	-0.01
Cash	-7.78	-0.79	-2.55	-0.36	0.14	0.00	0.86	0.01

This table shows the mean optimal allocation in percentage points to stocks, bonds and cash of a myopic investor, whose time horizon is one month, and the mean hedging optimal allocation in percentage points to stocks, bonds and cash of a strategic investor that is infinitely long lived. The optimal hedging allocation to stocks, bonds and cash is defined as the difference between the strategic investor asset demand that is infinitely long lived and the myopic investor asset demand, whose time horizon is one month. The optimal portfolio rule is specified in equation (5) and optimized for a power utility function with different CRRA coefficients (γ=2, 5, 20, 100) and a value of δ=0.95, using the detrended short-term interest rate (T_b), the U.S. credit spread (Def), the S&P 500 trend (Trend) and the one-month average of the excess stock and bond returns (ARP) as state variables. We use monthly data from January 1980 to December 2010.

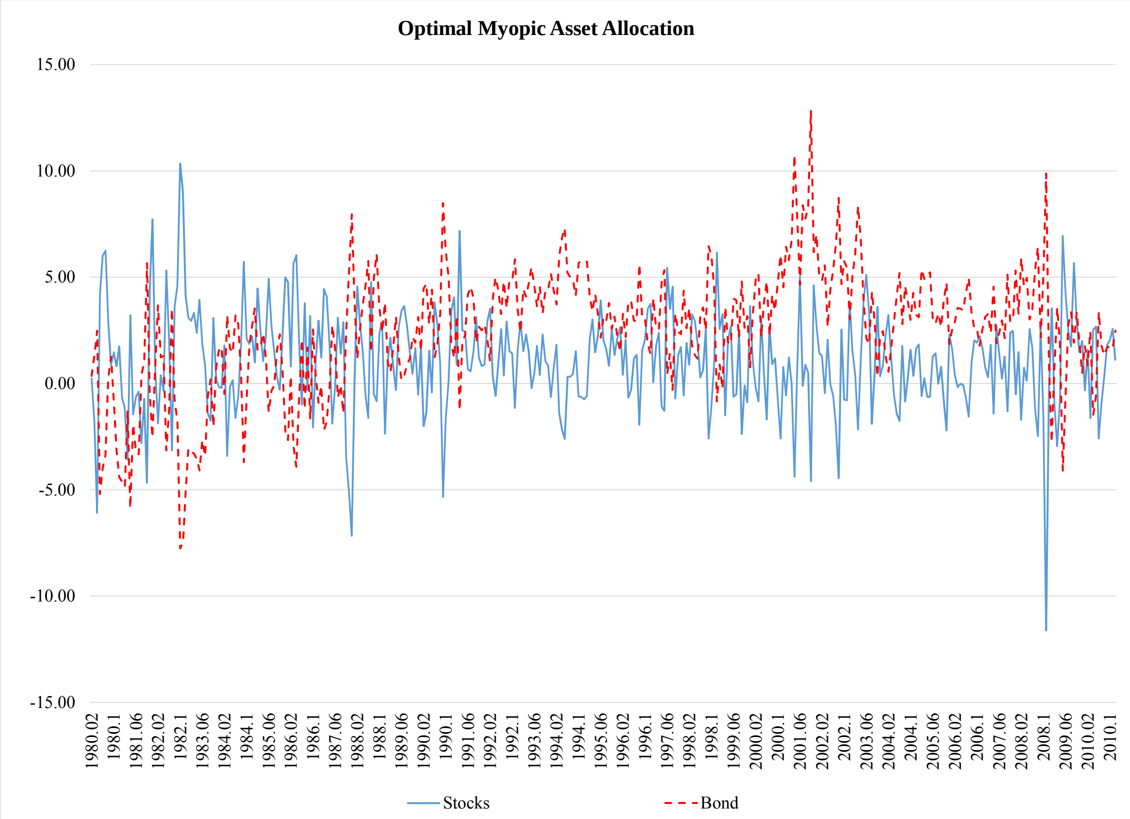

Figure.1: Optimal myopic allocation

This figure plots the optimal myopic allocation to stocks and bonds. The optimal portfolio rule is specified in equation (5) and optimized for a power utility function with a CRRA coefficient γ=5 and a value of δ=0.95, using the detrended short-term interest rate (T_b), the U.S. credit spread (Def), the S&P 500 trend (Trend) and the one-month average of the excess stock and bond returns (ARP) as state variables. We use monthly data from January 1980 to December 2010.

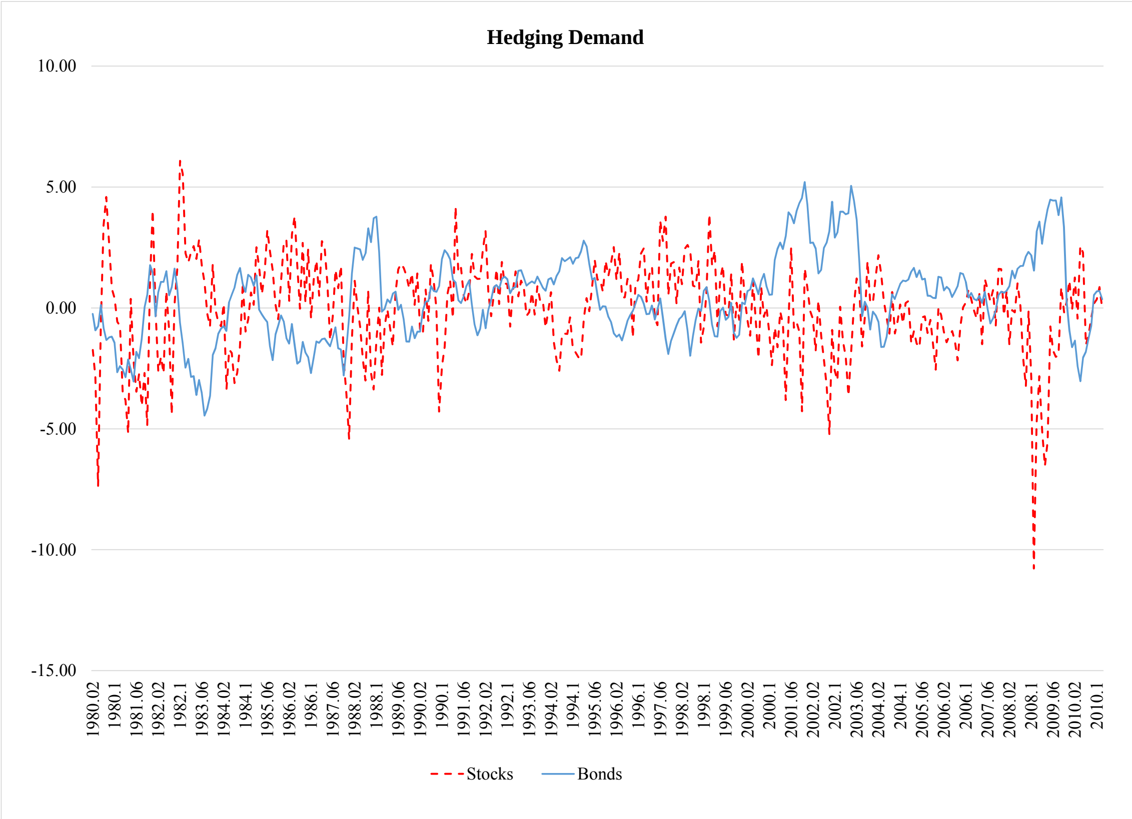

Figure.2: Optimal Hedging Demand

This figure plots the optimal hedging allocation to stocks and bonds defined as the difference between the investor strategic asset demand that is infinitely long lived and the myopic investor asset demand, whose time horizon is one month. The optimal portfolio rule is specified in equation (5) and optimized for a power utility function with a CRRA coefficient γ=5 and a value of δ=0.95, using the detrended short-term interest rate (T_b), the U.S. credit spread (Def), the S&P 500 trend (Trend) and the one-month average of the excess stock and bond returns (ARP) as state variables. We use monthly data from January 1980 to December 2010.